1 Introduction

Technology has increased tenfold over the past century, yet classrooms have found few ways to embrace these advances. Despite technology affecting almost every part of our life, teaching in many primary schools still operates in the same fundamental way that it did ten, twenty, or fifty years ago. The disconnect between technology and learning creates a problem for both students and teachers. Students miss out on the technological skills that they need today and in the future, while the teachers miss out on valuable ways to engage their students.

Technology has the ability to assist in alleviating many problems that are currently faced within today’s classrooms. Paper workbooks provided with textbooks allow little flexibility in the curriculum and are difficult for teachers to alter. Furthermore, as class sizes continue to grow larger, teachers have fewer opportunities to give one-on-one attention to their students. Classroom Learning Partner (CLP) will engage students with one-on-one interaction with teacher customized content by replacing science and math paper workbooks with tablet-based notebook versions.
This paper will introduce and describe CLP and its current place in the classroom. I will then discuss the stamp element of CLP and my contributions to its development as I attempt to create a machine intelligence around these objects.

2 Background

CLP operates in a classroom on a set of wirelessly connected tablet computers and is used as an electronic notebook. Students use CLP much as they would a paper notebook, writing and drawing in it. The notebook, however, can contain much more than a paper notebook, e.g., interactive and multimedia elements. It also enables students to wirelessly send their work to the teacher. Teachers use CLP to view the student work during class, identifying students who may need help and selecting examples of student work as the focus of class discussion.

2.1 CLP Interactions

Students interact with a personal CLP notebook on a tablet via a tablet pen or their finger, depending on the activity. By strictly having this type of interaction with no keyboard usage, CLP aims to preserve and take advantage of student familiarity with paper notebooks and drawing while also enhancing the possible activities with which a student can interact. The pen is especially important for the kinds of STEM (Science, Technology, Engineering, Math) activities that CLP targets, since many of these activities involve a mixture of text and drawing. Students may, for example, work a math problem and draw a picture to explain the answer, or collect and record data on the tablet, then graph the data. They submit their work wirelessly to the teacher, who can select examples to share and discuss with the class.

CLP provides a template within the notebook system that facilitates teachers creating their own student exercises. CLP aims to offer a richer teacher-student interaction experience by automatically analyzing student work submitted to the teacher and directing the teacher to
particularly interesting examples. This functionality will allow the teacher more time to focus on the students as a result of not needing to check and analyze every student submission.

2.2 Stamps

The stamp object is one of the interactive elements provided by CLP. A stamp allows a user to create a drawing containing multiple identical images. These identical images create a structured vocabulary that can aid interpretation, both by human and by machine. This paper will focus on the application of stamp objects to mathematical problems, as illustrated in Figure 1.

![Image of a fourth grade student using a stamp to solve a math fraction exercise](image1.png)

**Figure 1:** A fourth grade student using a stamp, on which she has drawn a balloon, to solve a math fraction exercise in her CLP notebook

A stamp object consists of an image and the parts, if any, represented by the image. There are two types of stamps used in CLP that differ only by the type of image: (1) a pre-drawn or image stamp, as shown in Figure 2, and (2) a blank stamp that enables a student to draw her own image, as shown in Figure 1. Image stamps allow teachers to scaffold student interactions with stamp objects by giving students examples of how stamps can be used to create pictures and construct explanations. Blank stamps create an opportunity for even deeper student engagement and an opportunity for student creativity and personal ownership while facilitating machine interpretation of student work. When stamps are used to teach arithmetic, information
about what a stamp represents is useful for automatic interpretation: The parts component of the stamp describes how many things are represented by the stamp. The bicycle shown in Figure 2, for example, represents an object with two wheels in this math problem. The stamp thus is representing a part-whole relationship that students are to use in working the problem.

![Figure 2: Drawing made using a pre-drawn stamp to solve a multiplication problem](image)

Stamp elements can be used to solve different types of arithmetic problems: addition, multiplication, division, and combination problems, e.g., that involve multiplication then addition of the resulting numbers. Such a combination problem, for example, might involve two stamps, where each stamp is used in an individual multiplication problem and then the resulting numbers are added together for a final answer. (See Figure 3.)
3 Problem

Evaluating and understanding student answers using stamps differs with each of the types of problems, but all of them depend on some combination of three basic properties: what the stamp represents, how many stamp images there are, and how the stamp images are arranged. Understanding the semantics of these properties for a drawing created using stamps can be difficult for both computer and human alike.

3.1 Stamp Representation

Understanding what the stamp means necessitates understanding how many things the stamp represents. This issue was briefly described in Section 2.2 when explaining the “parts” property of a stamp object. This information is used in multiplication and combination problems and is necessary in order for CLP to correctly “understand” what a student has drawn. In a problem involving groups of identical objects, for example, a student may choose to draw a single object on a stamp, or a group of the objects. In the example shown below in Figure 4, the
student chose to draw a single flower on the stamp rather than the five-flower bouquet mentioned in the problem.

![Division Stories](image)

**Figure 4: Representation made using student-drawn stamps to solve a division problem**

As suggested with the example in Figure 4, students may think of a variety of meanings for a single stamp in a word problem. Constraining the meaning of a stamp would enable more accurate machine interpretation, but would limit student creativity and exploration of multiple representations: Making the choice of a stamp for the student would both hinder learning, by giving them answers to parts of the problem, and by preventing students from thinking of alternate correct representations. As Figure 4 illustrates this student chose to create a stamp representing one flower and use fifteen instances of the stamp in her drawing; another student could have chosen to answer the same problem by creating a stamp representing an entire bouquet of five flowers and creating three instances of the stamp.

### 3.2 Number of Stamp Images

Knowing the number of stamp images means understanding how many times the stamp has been "stamped", creating a copy of its image onto the page. Knowing the number of stamps is needed for every type of stamp problem. Realizing how many stamp images exist can be a
trivial step for both humans and computers who both need only to count the objects to complete this piece of stamp understanding.

The most difficult case occurs when multiple stamp objects are being used or are believed to be being used. Currently a stamp image within the CLP platform does not know which from stamp it originated. Only through either human analysis or computer image analysis can the computer solve a multi-stamp problem. In addition, if a student edits a stamp after using it and believes it now represents a different stamp because it displays a different image, the computer analysis will be incorrect.

3.3 Stamp Image Arrangement

Some problems require students to group their stamp objects strategically in order to graphically display their thought process and illustrate their answer. This property is fundamental to comprehending division problems, as is seen in Figure 4. Unfortunately, understanding the stamp images’ arrangement is the most difficult property for a computer to analyze correctly and requires the computer to display the most “intelligence” out of all of the properties. This is a consequence of the loose structure of the property and the multiple forms of portrayal. Three basic structures can be used to form groupings: relational location, a line connecting the stamps in each grouping, and outlining the groups. Not only do each of these grouping techniques have their own individual difficulties, but there is also the added challenge of detecting which grouping technique is being used.

3.3.1 Relational Location

Relational location as a grouping mechanism relies on the user’s or computer’s understanding of space as a separating factor between objects, as depicted in Figure 5. This can be difficult for both computers and humans to assess if groups are not tightly packed together or one object strays dramatically from the group, which can result in inaccurate
grouping categorizations. The technical challenge comes from no guaranteed basic structure around which to build an intelligence. Every user can use a different spacing between groups, and groups within the same page may use a different spacing between objects and groups. Unlike the other two grouping mechanisms, relational location does not depend on the stroke object (a pencil or marker marking) for assistance.

![Spatial grouping representation of division problem](image)

**Figure 5: Spatial grouping representation of division problem**

### 3.3.2 Line

Connecting all objects in a group via a line is another type of grouping mechanism. As shown in Figure 6, this method must have a line going through all grouped objects to show their connection. The line has no restrictions and can be straight, horizontal, vertical, bent, jagged, or any other shape that involves two distinct end points. Some examples are depicted in Figure 7. This method can cause erroneous results as a result of stray marks, such as the two horizontal lines in Figure 6. In this case, a computer analysis of the page would result in either an incorrect grouping or an inability to analyze the problem detecting an error on the page because four of the stamps would appear twice in the groupings detected.
3.3.3 Outlining

Outlining is the most common grouping technique used by students. Groupings are marked by drawing a shape around all of the grouped objects. Generally this shape is a circle, but as Figure 4 displays, no constant shape need be used within the page. Many technical difficulties occur when trying to detect an outlining. First, the stroke object cannot tell the difference between a connected line creating a shape and a line such as shown in Figure 7. This leaves it up to the Stamp Analyzer to determine what is an outlining and what is a line grouping. Second, an outlining may be made up of multiple separate stroke objects, which is caused by the user creating separate visual lines resulting in one object visually. For instance, a square generally takes four separate strokes to create, but a human views the object as a
square, not four separate stray strokes. Third, as the line between the top and middle grouping in Figure 8 depicts, a user may use one stroke to act as a connecting stroke element for two separate groupings. Fourth, a stroke intending to act as an outlining element may go through multiple stamp images, confusing the analyzer whether the page is using outlining or line techniques to differentiate the objects into groups. The top and middle groupings in Figure 8 could be considered accurately as using line or outlining techniques to display the group. If the analyzer decided that a line technique was being used, it would erroneously not detect the third grouping. Finally, a group may go off the page as viewed in the bottom grouping of Figure 8. This creates a noncontinuous enclosure for the stamps, breaking the rule of outlining and invalidating the outlining rules for the computer. However, a human could clearly understand that a grouping was taking place so the analyzer should attempt to handle this case.

![Division Problem](image)

**Figure 8: A division problem assisted by an outlining grouping technique**

### 4 Technical Approach

When work on this project began, there was no support built into the CLP platform to support any of the three properties (stamp representation, the number of stamp images, and stamp image arrangement) required to create a computer intelligence around the stamps. An architecture had to be created to support these properties and then placed via human inspection
on top of pre-existing math problem submissions that utilized the CLP stamp element to test the intelligence created.

4.1 Stamp Representation

Stamp representation is the most difficult property to coerce from the student correctly. Students must have a thorough understanding of the problem to recognize what the stamp represents. Since in some cases what the stamp depicts differs from what the stamp represents in terms of the problem, this can be a confusing process for students. For instance, in Figure 2, the stamp is an image of a bicycle, but the stamp represents two wheels. The chosen representation uses a button displaying a number with the term “Parts” following it to convey to the student where to input the parts of the stamp. Upon selecting the parts button, a dialog keypad appears to allow student input. This representation creates a compromise between requiring keyboard input and not having a clear computer-readable answers.

![Keypad](image)

Figure 9: A keypad used to input stamp parts

To create an architecture for comprehension of stamp representation, students are scaffolded with an example stating the number of parts, creating the stamp, and labeling the stamp object, as seen in Figure 2. This is followed by an evolution process of students receiving a stamp but having to identify the stamp parts and then the students having to create the stamp themselves and then identifying the parts of that stamp.
4.2 Number of Stamp Images

When one stamp is used, detecting the number of stamps is trivial. This can be done either by counting all of the stamp objects on a page and subtracting one since one stamp must be the original stamp or by searching through all of the stamps and keeping a count of all of the stamps that are not an original. Although the first method is simpler, the second method lends itself well to the more difficult task of detecting how many stamps exist when multiple stamps are used and the computer must understand how many of each of these stamps exists. In this case, the second method needs only to be augmented by keeping a count for each unique stamp type.

Collecting the number of stamps off of a page was possible before this project; however, there was no indication of which stamp a stamp image derived from. To fix this problem and allow full support for counting the number of stamps when multiple stamps are used, a unique ID (UID) was attached to each original stamp image. This UID was then given as a property to all stamp images spawning from it. This allowed full support for detecting the number of each stamp object.

Upon implementation of this feature, it was noted that the cases of addition, multiplication, multi-multiplication and division could operate without this additional property. This was a result of addition, multiplication, and multi-multiplication operating correctly by looping through the stamps, checking whether the stamp was an original stamp or a stamp image, and then adding the parts of the stamp to a count. Division only uses one stamp, and therefore, UID is not needed. It is believed that a stamp UID could be used in problems that haven’t been currently identified.

4.3 Stamp Image Arrangement

Detecting groupings and analyzing the stamp image arrangement is the most difficult property to make understandable to the analyzer. This is the only property that the analyzer may
be unable to deliver an answer for. Reasons that this may occur are that one consistent method
cannot be found on the page or if all of the groupings do not contain the same number of stamp
images when analyzing a division problem as this would break a fundamental division rule.

As previously discussed, there are three possible ways to depict groupings: using space
as a separator, connecting the stamps via a line, and outlining each group. After examination of
previous student work, it was decided that the first iteration of development would not support
space as a separator. Spacing is too sporadic between problems and a consistent metric cannot
be found to build an algorithm around. For this method to accurately work, the stamp
intelligence will need to use machine learning for assessment, which was deemed beyond the
scope of this project. This leaves the two methods that involve strokes for visible confirmation
of groupings. These two methods achieve better accuracy for the computer, assist humans
interpreters, and have students affirm visually what they are trying to perform.

The first challenge for the stamp analyzer is selecting which method the student used.
To detect this, the analyzer takes the collection of all of the stamps on the page and then loops
through the collection looking for stamp images. Once it finds a stamp image, the analyzer
checks to see if there are any strokes that go through the stamp by iterating through all of the
strokes. It then checks the coordinates of each pixel within each stroke to check if they are in
the area of the stamp image. If a stroke does go through a stamp, the analyzer believes it is
using the line method. It then iterates through the rest of the stamps and checks if any other
stamps belong in the group. Once the entire set of stamps has been looped through, the stamps
found to be in the group are taken out of the collection, and the analyzer begins the process
again with the next free stamp. If the next stamp at any point of this process does not have a
stroke going through it or if the type of problem is division and the groups do not contain the
same number of elements, the analyzer assumes it is in the wrong method and starts the entire
process over with the outlining method.
The line method can give many false results because while a line to a human viewer may not go through a stamp image, it may still go through the stamp image. The stamp image is a large box that may have additional white space surrounding what a human would claim is the image. Because of this, a student may place a stroke through the white space assuming it does not touch the stamp image. For this reason, the analyzer tries the outlining method after attempting the line method to try to decrease the number of false negatives or problems that give no result.

A few methods were considered when developing the outlining method’s algorithm. The first method naively assumed that an outlining would consist of a single stroke and that these outlining would be square or contain space between groupings. This method looks for the minimum and maximum x and y coordinates and then finds all stamps that fall within these coordinates. It was soon noticed that using any coordinate belonging to a stamp was too broad of a metric so this was tightened to a majority of the stamp’s area falling into the space. This method’s main pitfall is that groupings are not necessarily square or have enough space to not cause false negatives from overlapping maximum and minimum coordinates between groupings. Figure 8 is a prime example of this error, which would cause the entire problem to be evaluated incorrectly.

Realizing that using the maximum and minimum x and y coordinates of the outlining enclosure produced too many incorrect results, the next approach used a smaller area, attempting to handle the diagonal groupings that failed the previous method. A smaller area called a “boundary box” is created using the maximum and minimum y coordinate and the x difference between the two closest sides as seen in Figure 10. This method compares every stamp against every other stamp, searching for strokes that pass through both the upper and lower half of the boundary box. If two stamps exist between which no strokes pass through their boundary box, it is believed that these two stamps are in the same group.
5 Future Work

This project has begun to create a stamp intelligence, but much more work will be needed before it can be used in the classroom. The development of the project has been created around examples of eight student submissions covering five different problems. Much further student analysis and testing is needed to analyze how accurate the algorithms created are and continue to adapt them to decrease the number of false positives and negatives.

All analysis for this project was done post classroom operations. For CLP analysis of stamp objects to be a true resource to teachers, it must be able to perform this analysis during classroom sessions so that the teacher can give immediate feedback to students when it will be most useful. For this to be possible, a representation of correct answers inputted by the teachers during notebook creation must be present. A successful representation system will not only work with stamp objects but will be a standard representation used for all types of problems. It will be up to future work for this standard and simplified representation system to be created.

While performing the post-classroom analysis, the researcher was able to clean up the data in order to provide working examples to the stamp analyzer. For the program to work accurately in the classroom, fault tolerance needs to be built into the system. For instance, handling stray marks that might interfere with correct grouping analysis or eliminating overlapping stamps that appear to the human eye as one stamp. The computer still sees the overlapping stamps as two separate stamps, causing the number of stamps property to be
incorrectly assessed. Both of these errors occurred numerous times within the student submissions, interfering with the stamp analyzer’s analysis.

Although not a necessity for successful implementation of the stamp intelligence, other features would be nice to include in future iterations, such as supporting all three types of groupings. The fewer rules that students must understand and adapt to in order to use the CLP infrastructure, the better chance of adoption and correct use. If too many rules need to be adhered to in order for the intelligence discussed in this report to give correct results, the intelligence will in all likelihood be useless, producing too many false positives or negatives from students forgetting rules.

The current implementation of the stamp object restricts student interactions by requiring the use of a keypad to inform the stamp analyzer how many parts make up the stamp in question. Future work should eliminate this restraint and allow students to return keyboard-less tablet interaction by writing within a box and the CLP using handwriting recognition software to understand the answer.

6 Conclusion

Stamps have already proven themselves to be a useful tool in classrooms as an element within the CLP platform. This project is the first step in evolving stamps into the tool that they could be, facilitating learning and diminishing the workload placed on teachers needing to grade all pages. The work here has acted as a proof of concept that an analyzer can be implemented upon the stamp object, but it produces too many false positives and negatives to use in a classroom in its current form.